Joint MCMA and DD blind equalization algorithm with variable-step size

Doaa Ashmawy (1) Kevin Banovic (2) Esam Abdel-Raheem (3) Mohamed Youssif (4) Hala Mansour (1)
Mahmoud Mohanna (5)
(1) ECE Dept., Benha University, Cairo, Egypt
(2) ECE Dept., University of Toronto, Ontario, Canada
(3) ECE Dept., University of Windsor, Ontario, Canada
(4) ECE Dept., Al-Azhar University, Cairo, Egypt
(5) Seismology Dept., NRIAG, Cairo, Egypt

***Abstract*—A variable step size technique is applied to joint Modified Constant Modulus Algorithm (MCMA) and Decision-Directed (DD) equalization algorithm to speed up convergence with respect to the original algorithm. The same technique is used with joint CMA and DD algorithm and exhibits improved performance.**

Introduction

Blind equalization algorithms are used to remove the intersymbol interference (ISI) caused by the communication channel without the use of a trainig sequence. The constant modulus algorithm, CMA [1]-[3] is a popular sheme for blind equalization and it has the ability to converge prior to phase recovery. The modified constant modulus algorithm (MCMA) [4] improves the performance of CMA by obtainig low steady state mean-square-error (MSE) and it also eliminates the need for joint carrier phase recovery. Decision-directed (DD) algorithm requires the MSE to be lower than a specified threshold and can not be applied at the beginning of equalization.

Applying a joint cost function [5]-[6] which combines MCMA and DD methods helps decreasing the convergence time of blind adaptive algorithms. The step size parameter of any adaptive algorithm plays an important role in determining the convergence time and the steady-state MSE [7]-[10]. By applying a VSS technique to the joint cost function, the convergence rate is accelerated. Simulation results using SPIB microwave channels [11] with 16/64 QAM modulation scheme show the effectiveness of the proposed algorithm. Applying the same technique to joint CMA and DD also improves the performance of the algorithm.

I. BLIND EQUALIZATION

A. System Model

The baseband model of an adaptive blind channel equalization system is shown in fig.1. The received signal at the input of the equalizer $x(n)$ can be expressed as

$$x(n) = \sum_{k=0}^{L_h-1} h(k)s(n-k) + w(n), \qquad (1)$$

where $h(n)$ is the impulse response of the channel with length L_h. The input data sequence $s(n)$ is assumed to be independent, identically distributed (i.i.d.) random variable with zero mean and variance σ_s^2. $w(n)$ is assumed to be additive white gaussian noise (AWGN) with zero-mean and variance σ_w^2. The equalizer output can be expressed as

$$y(n) = \sum_{k=0}^{L_C-1} c_k(n)x(n-k) = \boldsymbol{X}^T(n)\boldsymbol{C}(n), \qquad (2)$$

where $\boldsymbol{C}(n) = [c_o(n), c_1(n), \ldots, c_{L_C-1}(n)]^T$ is the equalizer tap weights vector, $\boldsymbol{X}(n) = [x(n), x(n-1), \ldots, x(n-L_C+1)]^T$ is the equalizer input data vector and L_C is the length of the equalizer while superscript T stands for transpose of vector.

B. Constant Modulus Algorithm (CMA)

The CMA [4]-[6] is a popular scheme for blind equalization of QAM systems. The cost function of the CMA is phase-blind and the CMA can converge even when there is a phase error. The cost function of CMA has the form

$$\Psi_{CMA,p}(n) = \frac{1}{4}E[(|y(n)|^p - \gamma_{CMA,p})^2], \qquad (3)$$

For the special case of $p = 2$, the CMA cost function has the form

$$\Psi_{CMA}(n) = \frac{1}{4}E[(|y(n)|^2 - \gamma_{CMA})^2], \qquad (4)$$

where $E[.]$ denotes statistical expectation and γ_{CMA} is a positive real constant defined by

$$\gamma_{CMA} = \frac{E[|s(n)|^4]}{E[|s(n)|^2]}, \qquad (5)$$

The equalizer coefficient update is given by

$$\begin{aligned}\boldsymbol{C}(n+1) &= \boldsymbol{C}(n) - \mu\,\nabla\Psi_{CMA}(n)\\ &= \boldsymbol{C}(n) - \mu\, e_{CMA}\,\boldsymbol{X}^*(n), \qquad (6)\end{aligned}$$

and $e_{CMA} = y(n)(|y(n)|^2 - \gamma_{CMA})$

where ∇ denotes stochastic gradient of the CMA cost function $\Psi_{CMA}(n)$ with respect to the tap weights vector $\boldsymbol{C}(n)$, μ is the step-size parameter, $e_{CMA}$is the CMA error and the asterisk denotes complex conjugation.

Since the CMA cost function is carrier- phase independent, the equalizer output will have a constant phase rotation. To solve this problem, some techniques for joint blind equalization and carrier recovery [1], [12]-[13] were presented in the literature.

C. Modified CMA (MCMA)

By modifying the CMA cost function (4) in the form of cost functions for real and imaginary parts, the MCMA cost function is written as [4]

$$\Psi_{MCMA}(n) = \Psi_{MCMA,R}(n) + \Psi_{MCMA,I}(n), \tag{7}$$

where $\Psi_{MCMA,R}(n)$ and $\Psi_{MCMA,I}(n)$ are the real and imaginary parts of the cost function and are defined as [4]

$$\Psi_{MCMA,R}(n) = \frac{1}{4}E[(|y_R(n)|^2 - \gamma_{MCMA,R})^2], \tag{8}$$

$$\Psi_{MCMA,I}(n) = \frac{1}{4}E[(|y_I(n)|^2 - \gamma_{MCMA,I})^2], \tag{9}$$

where $y_R(n)$ and $y_I(n)$ are the real and imaginary parts of the equalizer output $y(n)$, i.e. $y(n) = y_R(n) + jy_I(n)$.

Assuming that the input data sequence $s(n) = s_R(n) + j = s_I(n)$ is i.i.d. random variable, the constants $\gamma_{MCMA,R}$ and $\gamma_{MCMA,I}$ can be defined as

$$\gamma_{MCMA,R} = \frac{E[|s_R(n)|^4]}{E[|s_R(n)|^2]}, \tag{10}$$

$$\gamma_{MCMA,I} = \frac{E[|s_I(n)|^4]}{E[|s_I(n)|^2]}, \tag{11}$$

The equalizer tap weights vector is updated according to the following equation

$$\begin{aligned}\boldsymbol{C}(\boldsymbol{n}+\boldsymbol{1}) &= \boldsymbol{C}(n) - \mu\, \nabla\Psi_{MCMA}(n) \\ &= \boldsymbol{C}(n) - \mu\, e_{MCMA}(n)\boldsymbol{X}^*(n),\end{aligned} \tag{12}$$

where the error signal $e_{MCMA}(n) = e_{MCMA,R}(n) + je_{MCMA,I}(n)$ is given by

$$e_{MCMA,R} = y_R(n)(|y_R(n)|^2 - \gamma_{MCMA,R}), \tag{13}$$

$$e_{MCMA,I} = y_I(n)(|y_I(n)|^2 - \gamma_{MCMA,I}), \tag{14}$$

The MCMA can sufficiently recover an arbitrary phase shift and track the carrier frequency offset to some extent.

II. DECISION DIRECTED (DD) ALGORITHM

The cost function of the DD algorithm can be described as [13]

$$\Psi_{DD}(n) = \frac{1}{2}E[(y(n) - \hat{y}(n))^2], \tag{15}$$

where $\hat{y}(n)$ is the value of the decided output as shown in Fig.1.

The complex equalizer coefficient update is written as

$$\begin{aligned}\boldsymbol{C}(n+1) &= \boldsymbol{C}(n) - \mu\, \nabla\Psi_{DD}(n) \\ &= \boldsymbol{C}(n) - \mu\, \hat{e}(n)\boldsymbol{X}^*(n),\end{aligned} \tag{16}$$

where $\hat{e}(n) = y(n) - \hat{y}(n)$ is the value of the estimated error. As evident from (16), the equalizer coefficients updates depend on the difference between the desired output and the equalizer output.

III. VARIABLE STEP SIZE (VSS) BLIND EQUALIZATION

In [7], a VSS technique was developed and applied to CMA for 16 QAM signals. The equalizer coefficients are updated accoring to the following relation [7]

$$\boldsymbol{C}(n+1) = \boldsymbol{C}(n) - \mu(n)e(n)\boldsymbol{X}^*(n)$$

where $e(n)$is the error signal and

$$\mu(n+1) = \frac{\mu(n)}{1+\lambda\mu(n)|e(n)|^2}, \tag{17}$$

The parameter λ is defined as

$$\lambda = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \begin{array}{l}\text{if } sgn(Re\{e(n)\}) = sgn(Re\{e(n-1)\}) \\ \text{and } sgn(Im\{e(n)\}) = sgn(Im\{e(n-1)\})\end{array} \\ 1 & \text{otherwise,} \end{cases} \tag{18}$$

with $sgn\{.\}$ denotes signum function.

IV. PROPSED ALGORITHM

A. Joint MCMA-DD with VSS

By using a joint cost function, which can be expressed as the sum of the cost functions of MCMA and DD algorithms [5], the resulting blind equalization algorithm can have a fast convergence rate. This joint cost function is described by [5]

$$\begin{aligned}\Psi_{MCMA-DD}(n) &= \frac{1}{4}E[(|y(n)|^2 - \gamma_{MCMA})^2] \\ &+ \frac{1}{4}E[(|y(n)|^2 - |\hat{y}(n)|^2)^2],\end{aligned} \tag{19}$$

The equalizer coefficients update equation can be described by

$$\boldsymbol{C}(n+1) = \boldsymbol{C}(n) - \mu\, \nabla\Psi_{MCMA-DD}(n)$$

$$= \boldsymbol{C}(n) - \mu\, e_{MCMA-DD}(n)\, \boldsymbol{X}^*(k), \qquad (20)$$

where the error signal, $e_{MCMA-DD}\,(n) = e_{MCMA-DD,R}(n) + je_{MCMA-DD,I}(n)$ is given by

$$e_{MCMA-DD,R} = y_R(n)\big(|y_R(n)|^2 - \gamma_{MCMA,R}\big) + y_R\big(y_R^2(n) - \hat{y}_R^2(n)\big), \qquad (21)$$

and

$$e_{MCMA-DD,I} = y_I(n)\big(|y_I(n)|^2 - \gamma_{MCMA,I}\big) + y_I\big(y_I^2(n) - \hat{y}_I^2(n)\big), \qquad (22)$$

To integrate the VSS technique, we can modify (20) to be of the form

$$\boldsymbol{C}(n+1) = \boldsymbol{C}(n) - \mu(n) e_{MCMA-DD}(n)\, \boldsymbol{X}^*(k), \qquad (23)$$

and the VSS $\mu(n)$ can be described as in (17) with $e(n) = e_{MCMA-DD}$ given by (21) and(22).

B. Joint CMA-DD with VSS

The CMA-DD joint cost function can be expressed as

$$\Psi_{CMA-DD}(n) = \frac{1}{4}E[(|y(n)|^2 - \gamma_{CMA})^2] + \frac{1}{4}E[(|y(n)|^2 - |\hat{y}\,(n)|^2)^2], \qquad (24)$$

The VSS equalizer coefficients are updated according to

$$\boldsymbol{C}(n+1) = \boldsymbol{C}(n) - \mu(n)\mu\, \nabla\Psi_{CMA-DD}(n)$$

$$= \boldsymbol{C}(n) - \mu(n)\, e_{CMA-DD}(n)\, \boldsymbol{X}^*(k), \qquad (25)$$

where the error signal $e_{CMA-DD}\,(n)$ is given by

$$e_{CMA-DD} = y(n)(|y(n)|^2 - \gamma_{CMA}) + y(n)\big(y^2(n) - \hat{y}^2(n)\big), \qquad (26)$$

And $\mu(n)$ is given by (17) with $e(n) = e_{CMA-DD}$ given by (26).

V. SIMULATIONS

Simulations were carried out in a 35-dB SNR environment with 16- and 64-QAM for MCMA with a step-size $\mu = 2^{-10}$, joint MCMA-DD with a step-size $\mu = 2^{-10}$, and joint MCMA-DD with VSS where the initial step-size $\mu_i = 2^{-7}$. The channels were $T/2$- spaced microwave channels from SPIB (#2) [11], where T is the symbol period. The equalizers were 16-tap -spaced finite impulse response (FIR) filters, which were initialized by a unitary double center spike. Simulation results are illustrated for SPIB microwave channel #2 in Fig. 2 and 3 for 16- and 64-QAM, respectively. The MSE was calculated as the average instantaneous-squared error across the decision block over 200 realizations. On average, joint MCMA-DD with VSS achieves faster convergence time than either MCMA or joint MCMA-DD with fixed step size.

The same observations are made for CMA with a step-size $\mu = 2^{-10}$, joint CMA-DD with a step-size $\mu = 2^{-10}$, and joint CMA-DD with VSS where the initial step-size $\mu_i = 2^{-7}$.The simulations are shown in Figs. 4 and 5 for 16- and 64- QAM, respectively. For 64- QAM and CMA equalization, the phase recovery is implemented jointly after equalization for the results to comparable to joint CMA-DD and joint CMA-DD with VSS.

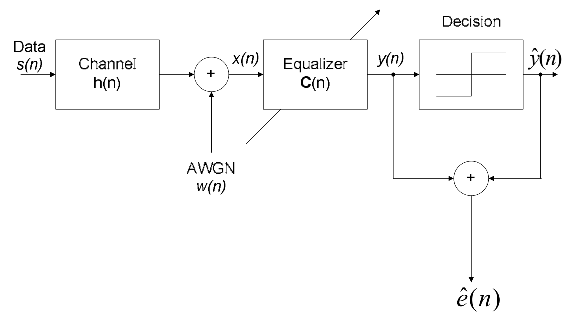

Figure 1. Baseband model of an adaptive blind channel equalization system

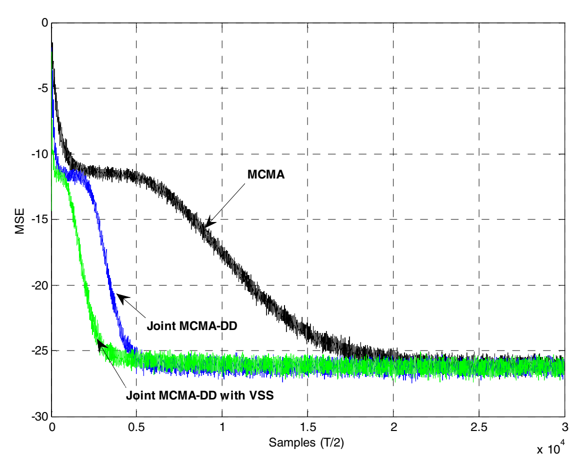

Figure 2. MCMA-based algorithms simulation results for 16-QAM using SPIB microwave chan #2.

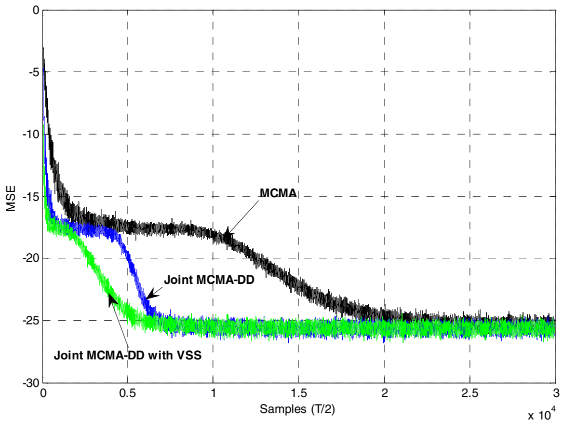

Figure 3. MCMA-based algorithms simulation results for 64-QAM using SPIB microwave chan #2.

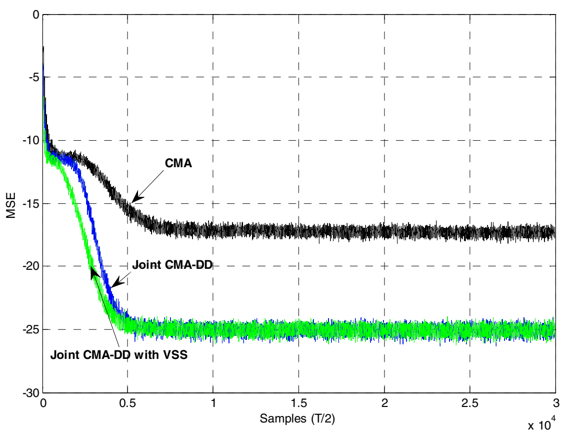

Figure 4. CMA-based algorithms simulation results for 16-QAM using SPIB microwave chan #2.

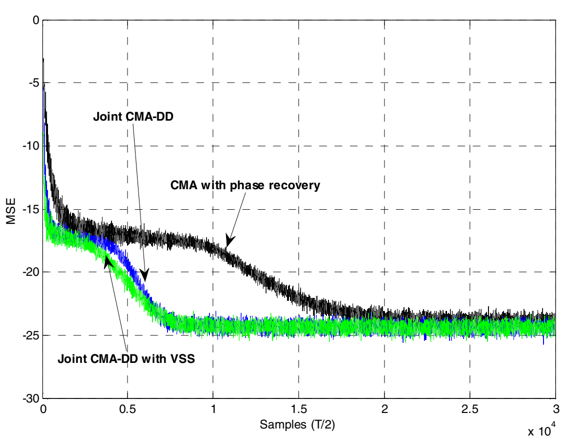

Figure 5. CMA-based algorithms simulation results for 64-QAM using SPIB microwave chan #2.

VI. Conclusion

We have integrated a VSS technique with joint MCMA-DD or CMA-DD algorithms and the resulting algorithms provide faster convergence than either the original algorithms or the CMA or MCMA adaptive blind equalization schemes. The simulation reults show the effectiveness of the VSS in enhancing the convergence speed of the algorithms.

References

[1] D.N. Godard, “Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communication Systems,” IEEE Trans. Commun., vol. COM-28, pp. 1867-1875, Nov. 1980.

[2] J.R Treichler and B.G.Agee, “A New Approach to Multipath Correction of Constant Modulus Signals,’” IEEE Trans. Acoust. , Speech, Signal Processing, vol. ASSP-31, pp. 459-472, Apr. 1983.

[3] J.R Treichler and M.G Larimore, “New Processing Techniques Based on the Constant Modulus Algorithm,” IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-33, pp. 420-431, Apr. 1985.

[4] K.N. Oh and Y.O. Chin, “Modified Constant Modulus Algorithm: Blind Equalization and Carrier Phase Recovery Algorithm,” Proc. 1995 IEEE Int. Conf. Comm., vol. 1, Seattle, WA, pp. 498-502, June 1995.

[5] Chih- Peng Fan, Wen- Hsuan Liang and Wei Lee,” Fast blind equalization with two-stage single/multilevel modulus and DD algorithm for high order QAM cable systems”, ISCAS 2008 , Seattle, WA, pp. 3001-3004, 2008

[6] Chih-Peng Fan, Wen-Hsuan Liang, and Wei Lee, “Efficient fast blind equalization with two- stage single/ multilevel modulus and DD algorithm for 64/256/1024 QAM wired cable communications”, Journal of the Chinese Institute of Engineers, Vol. 32, No. 1, pp. 1-15, 2009.

[7] E. Abdel-Raheem, "Blind Adaptive Equalization with Variable Step Size," ICICT '06,Cairo, Egypt, pp.91-100, Dec.2006.

[8] T. Aboulnasr and K. Mayyas, “A robust variable step-size LMS algorithm: analysis and simulation,” IEEE Trans. Signal Processing, vol. 45, no. 3, pp. 631-639, March 1997

[9] Z. Ramadan and A. Poularikas, “Performance analysis of a new variable step-size LMS algorithm with error nonlinearities,” Proc. IEEE SSST04, Atlanta, Georgia, pp. 384-388, Mar.2004.

[10] K. Banovic, E. Abdel-Raheem, and M.A.S. Khalid, “A novel radius-adjusted approach for blind adaptive equalization,” IEEE Signal Processing Lett., vol. 13, No.1, pp. 37-40, Jan. 2006.

[11] Signal processing information base (SPIB), http://spib.rice.edu

[12] A. Benveniste and M. Goursat, “Blind Equalizers,” IEEE Trans. Commun., vol. COM-32, pp. 871-883, Aug. 1984.

[13] G. Picchi and G. Prati, “Blind Equalization and Carrier Recovery Using a “Stop-and-Go” Decision-Directed Algorithm,” IEEE Trans. Commun., vol. COM-35, pp. 877-887, Sep. 1987.